Physics Letters B 760 (2016) 324–328

Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Ring wormholes via duality rotations

Gary W. Gibbons [a,b], Mikhail S. Volkov [b,c,*]

[a] DAMTP, University of Cambridge, Wilberforce Road, Cambridge CB3 0WA, UK
[b] Laboratoire de Mathématiques et Physique Théorique, LMPT CNRS – UMR 7350, Université de Tours, Parc de Grandmont, 37200 Tours, France
[c] Department of General Relativity and Gravitation, Institute of Physics, Kazan Federal University, Kremlevskaya street 18, 420008 Kazan, Russia

ARTICLE INFO

Article history:
Received 27 June 2016
Accepted 5 July 2016
Available online 9 July 2016
Editor: M. Cvetič

ABSTRACT

We apply duality rotations and complex transformations to the Schwarzschild metric to obtain wormhole geometries with two asymptotically flat regions connected by a throat. In the simplest case these are the well-known wormholes supported by phantom scalar field. Further duality rotations remove the scalar field to yield less well known vacuum metrics of the oblate Zipoy–Voorhees–Weyl class, which describe ring wormholes. The ring encircles the wormhole throat and can have any radius, whereas its tension is always negative and should be less than $-c^4/4G$. If the tension reaches the maximal value, the geometry becomes exactly flat, but the topology remains non-trivial and corresponds to two copies of Minkowski space glued together along the disk encircled by the ring. The geodesics are straight lines, and those which traverse the ring get to the other universe. The ring therefore literally produces a hole in space. Such wormholes could perhaps be created by negative energies concentrated in toroidal volumes, for example by vacuum fluctuations.

Wormholes are bridges or tunnels between different universes or different parts of the same universe. They were first introduced by Einstein and Rosen (ER) [1], who noticed that the Schwarzschild black hole actually has two exterior regions connected by a bridge. The ER bridge is spacelike and cannot be traversed by classical objects, but it has been argued that it may connect quantum particles to produce quantum entanglement and the Einstein–Pololsky–Rosen (EPR) effect [2], hence ER=EPR [3]. Wormholes were also considered as geometric models of elementary particles – handles of space trapping inside an electric flux, say, which description may indeed be valid at the Planck scale [4]. Wormholes can also describe initial data for the Einstein equations [5] (see [6] for a recent review) whose time evolution corresponds to the black hole collisions of the type observed in the recent GW150914 event [7].

An interesting topic is traversable wormholes – globally static bridges accessible for ordinary classical particles or light [8] (see [9] for a review). In the simplest case such a wormhole is described by a static, spherically symmetric line element

$$ds^2 = -Q^2(r)dt^2 + dr^2 + R^2(r)(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2), \tag{1}$$

where $Q(r)$ and $R(r)$ are symmetric under $r \to -r$ and $R(r)$ attains a non-zero global minimum at $r = 0$. If both Q and R/r approach unity as $r \to \pm\infty$ then the metric describes two asymptotically flat regions connected by a throat of radius $R(0)$. The Einstein equations $G^\mu_\nu = T^\mu_\nu$ imply that the energy density $\rho = -T^0_0$ and the radial pressure $p = T^r_r$ satisfy at $r = 0$

$$\rho + p = -2\frac{R''}{R} < 0, \qquad p = -\frac{1}{R^2} < 0. \tag{2}$$

It follows that for a static wormhole to be a solution of the Einstein equations, the Null Energy Condition (NEC), $T_{\mu\nu}v^\mu v^\nu = R_{\mu\nu}v^\mu v^\nu \geq 0$ for any null v^μ, must be violated. Another demonstration [8] of the violation of the NEC uses the Raychaudhuri equation [10] for a bundle of light rays described by θ, σ, ω: the expansion, shear and vorticity. In the spherically symmetric case one has $\omega = \sigma = 0$ [9], hence

$$\frac{d\theta}{d\lambda} = -R_{\mu\nu}v^\mu v^\nu - \frac{1}{2}\theta^2 . \tag{3}$$

If rays pass through a wormhole throat, there is a moment of minimal cross-section area, $\theta = 0$ but $d\theta/d\lambda > 0$, hence $R_{\mu\nu}v^\mu v^\nu < 0$ and the NEC is violated.

If the spacetime is not spherically symmetric then the above arguments do not apply, but there are more subtle geometric considerations showing that the wormhole throat – a compact two-surface of minimal area – can exist if only the NEC is violated [11,12]. As a result, traversable wormholes are possible if only the energy density becomes negative, for example due to vacuum

* Corresponding author.
E-mail addresses: gwg1@cam.ac.uk (G.W. Gibbons), volkov@lmpt.univ-tours.fr (M.S. Volkov).

http://dx.doi.org/10.1016/j.physletb.2016.07.012